THERMAL RADIATION EFFECTS ON PERIODIC MHD NATURAL CONVECTION FLOW ALONG A VERTICAL SURFACE

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ABSTRACT
The MHD natural convection periodic boundary layer flow of an electrically conducting and optically dense gray viscous incompressible fluid along a heated vertical plate is analyzed. The aim of this study is to observe the influence of transverse sinusoidal magnetic field which has its applications in induction electromagnetic pumps as well as in heat exchanger devices. Two numerical techniques are adopted to obtain the solution of the problem namely, (i) Direct numerical simulation (DNS) method which is used to integrate the governing equations directly (ii) Keller-box method which is applied after converting the governing equations into a convenient dimensionless form; using stream function formulation (SFF). For the entire range of X, numerical results are demonstrated graphically by showing the influence of important physical parameters, namely, the magnetic field parameter, M, Planck constant, R_d and the surface temperature parameter, θ_w, in terms of local skin friction coefficient and local Nusselt number coefficient as well as in terms of streamlines and isotherms; particularly for the case of liquid metals.

Keywords: Thermal Radiation, Magnetohydrodynamics, Natural Convection.

1. INTRODUCTION

Turcotte and Lyons [1] initially considered the periodic form of MHD boundary layer flow between two parallel walls. They also showed that inviscid theory may be used to calculate the overall performance of electromagnetic pumps and generators while the boundary layer theory may be used to obtain the wall shear stress. In order to obtain the solution, they adopted regular perturbation method up to first order correction for larger values of magnetic interaction parameter M^2 $(= \sigma B_0^2 \lambda / \rho U_r$, where U_r is the reference velocity of the fluid at the outer edge of the momentum boundary layer). Further, to verify the solutions obtained from series solution method, they also given consideration to Karman-Pohlhausen integral method.

Since thermal radiation effects are important in context of space technology and processes involving high temperatures therefore Ozisik [2], Sparrow and Cess [3] and Arpaci [4] initially studied the interaction of thermal radiation and natural convection. Later, considering the Rosseland diffusion approximation, investigations on the natural convection flow as well as on the mixed convection flow of an optically dense gray viscous fluid past or along heated bodies of different geometries have been accomplished by Hossain *et al.*

[5]-[6], Hossain and Munir [7], Hossain and Rees [8], Molla and Hossain [9], Siddiqa *et al.* [10].

In this work, natural convection flow along a semi-infinite vertical plate of electrically conducting and optically dense gray fluid is considered in the presence of sinusoidal transverse magnetic field (see Turcotte and Lyons [1]). The aim of this study is to look at the effects of transverse sinusoidal magnetic field which has its applications in induction electromagnetic pumps; since magnetohydrodynamic power generators also use induction designs. The boundary layer equations are solved numerically with two different methods. On one hand direct numerical simulation method is applied to discover the flow pattern; while on the other hand governing equations are reduced to convenient form by the introduction of the stream function formulation (SFF). The non-similar equations obtained from the SFF are solved by Keller-box method for the whole range of X that measure the axial distance from the leading edge. Numerical results thus obtained are expressed graphically in terms of local skin friction and local Nusselt number coefficients with effect of physical parameters, such as the magnetic field parameter, M, thermal radiation parameter, R_d and surface temperature parameter, θ_w for the fluids having Prandtl number

Pr=0.05. Effect of sinusoidal magnetic field on streamlines and isotherm lines have also been discussed and shown graphically.

2. MATHEMATICAL FORMULATION

Consider the steady 2D natural convection flow of a viscous, electrically conducting and optically dense gray fluid along a semi infinite vertical heated surface in the presence of magnetic field prescribed in the streamwise direction, $B_{\bar{y}}(\bar{x})$, following Turcotte and Lyons [1], is given as

$$B_{\bar{x}} = 0, \quad B_{\bar{y}} = B_0 \sin\left(\frac{\pi\bar{x}}{\lambda}\right). \tag{1}$$

where B_0 and λ are the constants related to transverse magnetic field and wavelength of the applied magnetic field, respectively. From the relation given in (1) it is depicted that induced magnetic field is not considered and induced fields can be neglected if the magnetic Reynolds number ($R_m = \mu_0 \sigma \bar{u} \lambda$, where μ_0 is the magnetic permeability of the free space) is much less than unity. The devices which are functional due to liquid metals usually have magnetic Reynolds number between 0.1 to 0.01. If the induced magnetic field effects are negligible then the effects of applied field are obtained by solving Laplace's equation in the boundary layer region. Its worthy to mention here that Turcotte and Lyons [1] were the first to consider the magnetic field of periodic form as given in (1), which has special applications in the induced electromagnetic pumps and generators. It is assumed that the surface temperature, T_w, of the flat plate is higher than the ambient fluid temperature, T_∞. Further, all the thermo-physical fluid properties are considered to be constant with negligible viscous dissipation effects.

Thus the fundamental equations under the usual Boussinesq approximation for steady magnetohydrodynamic flow with Ohm's law and Maxwell's equations associated with thermal radiation may now be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho}\sin^2\left(\frac{\pi\bar{x}}{\lambda}\right)\bar{u} + g\beta(T - T_\infty) \tag{3}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\kappa}\frac{\partial q_r}{\partial \bar{y}}\right) \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are the $\bar{x}$ and $\bar{y}$-components of the velocity field, respectively, in the momentum boundary layer; T is the temperature of the fluid in the thermal boundary layer, ρ the density of the fluid, ν the kinematic coefficient of viscosity, $-g$ identifies the gravitational vector, β the coefficient of thermal expansion, κ the thermal conductivity and α the thermal diffusivity.

The term q_r in energy equation represents radiative heat flux in the $\bar{y}$-direction. In order to reduce the complexity of the problem and to provide a means of

comparison with further studies that might employ a more detailed representation for the radiative heat flux, here the optically thick radiation limit, known as Rosseland diffusion approximation (see Ozisik [2]), is considered. Due to this assumption, the radiative heat flux q_r is as given in (5).

$$q_r = -\frac{4\sigma_0}{3(a + \sigma_s)}\frac{\partial T}{\partial \bar{y}} \tag{5}$$

In Eq. (5) 'a' is the Rosseland mean absorption coefficient, σ_s is the scattering coefficient and σ_0 is the Stefan-Boltzmann constant. Diffusion approximation is valid in the interior of a medium but not employed near the boundaries and is good only for intensive absorption, that is, for an optically thick boundary layer. The coordinate system and the flow configuration of the problem are shown in Fig. 1.

The boundary conditions to be satisfied are

$$\bar{y} = 0: \quad \bar{u} = \bar{v} = 0, \quad T = T_w \tag{6}$$

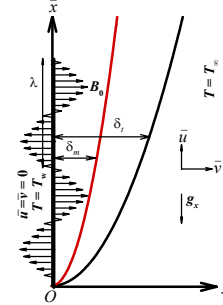$$\bar{y} = \infty: \quad \bar{u} = 0, \quad T = T_\infty$$

Fig. 1 Physical model and coordinate axes

In order to obtain the governing equations in dimensionless dependent and independent variables, we introduce the following parameters.

$$\bar{u} = \frac{\nu}{\lambda}Gr_\lambda^{\frac{1}{2}}u, \quad \bar{v} = \frac{\nu}{\lambda}Gr_\lambda^{\frac{1}{4}}v, \quad x = \frac{\bar{x}}{\lambda},$$

$$y = \frac{\bar{y}}{\lambda}Gr_\lambda^{\frac{1}{4}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr_\lambda = \frac{g\beta(T_w - T_\infty)\lambda^3}{\nu^2} \tag{7}$$

where Gr_λ is the Grashof number which measures the ratio of buoyancy force to the viscous force. Substituting (7) in (2)-(6), we get the following set of governing equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2\sin^2(\pi x)u + \theta \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial}{\partial y}\left\{\left(1 + \frac{4}{3}R_d(1 + \Delta\theta)^3\right)\frac{\partial \theta}{\partial y}\right\} \tag{10}$$

The corresponding boundary conditions are

$$y = 0: \qquad u = v = 0, \quad \theta = 1 \qquad (11)$$

$$y \to \infty: \qquad u \to 0, \quad \theta \to 0$$

Where

$$M^2 = \frac{\sigma B_0^2 \lambda^2}{\rho \nu Gr_\lambda^{1/2}}, \quad R_d = \frac{4\sigma_0 T_\infty^3}{\kappa(a + \sigma_s)}, \quad \Delta = \theta_w - 1 \qquad (12)$$

In Eq. (12) M is the magnetic interaction parameter (or simply, the magnetic parameter), that controls the strength of the sinusoidal magnetic field, R_d is the Planck constant or thermal radiation parameter which acts as a heat source due to the thermal radiation, θ_w denotes the surface temperature parameter that measures the ratio of the surface temperature to the ambient fluid temperature and Pr symbolizes the Prandtl number which gives the ratio of momentum diffusivity to the thermal diffusivity.

Now we draw our attention in finding the solution of equations (8)-(11) with the direct numerical simulation (DNS) method. For this purpose Eqs. (8)-(11) are discretized with the help of difference quotients. The partial derivatives in y are replaced with central difference formula while backward difference quotients are introduced in place of derivatives in x direction. Conclusively, we get the discretized equations with unknown's u, θ and v. These coupled equations are solved independently with the aid of tri-diagonal solver. The computation has been started at x=0.01 and then it marches up to x=10.0 using the step length Δx=0.01. In the computation procedure, the accuracy of 10^{-6} is achieved. Numerical experiments show that the maximum value of y which gives precise results is 20.0 with Δy=0.01. Thus, physically important quantities, namely, coefficient of local skin friction, $C_{f\lambda}Gr_\lambda^{-3/4}$, and coefficient of local Nusselt number, $Nu_\lambda Gr_\lambda^{-1/4}$, are measured from the following relations

$$C_{f\lambda}Gr_\lambda^{-3/4} = \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$Nu_\lambda Gr_\lambda^{-1/4} = -\left(1 + \frac{4}{3}R_d \theta_w^3\right)\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \qquad (13)$$

and entered in Tables 1 against X [0.01,10.0] for R_d =0.0 and 2.0 while values of other parameters are taken as $\theta_w = 2.1$, M=0.5 and Pr=0.05.

3. STREAM FUNCTION FORMULATION (SFF)

In this section, we introduce the stream function formulations (SFF), in order to reduce the boundary layer equations (8)-(11) to a system of parabolic equations which are then integrated by applying Keller-box method. The transformations for the stream function formulation are:

$$\psi = X^{\frac{3}{4}}(1+X)^{-\frac{1}{4}} f, \quad \eta = \frac{y(1+X)^{\frac{1}{4}}}{X^{\frac{1}{4}}}, \quad x = X, \quad \theta = \Theta \quad (14)$$

where ψ is the stream function which satisfies the equation of continuity given in (8). Introducing (14) into the boundary layer equations (8)-(11), we acquire the following non-similar equations

$$f''' + \frac{(3+2X)}{4(1+X)} ff'' - \frac{1}{2(1+X)} f'^2 - M^2 X^{1/2}(1+X)^{1/2}\sin(\pi X) f'$$

$$+(1+X)\theta = X\left(f'\frac{\partial f'}{\partial X} - f''\frac{\partial f}{\partial \xi}\right) \qquad (15)$$

$$\frac{1}{\text{Pr}}\left\{\left(1 + \frac{4}{3}R_d(1+\Delta\Theta)^3\right)\Theta'\right\}' + \frac{(3+2X)}{4(1+X)} f\Theta' = X\left(f'\frac{\partial\theta}{\partial X} - \theta'\frac{\partial f}{\partial X}\right) \quad (16)$$

The boundary conditions to be satisfied are

$$f(\xi,0) = f'(\xi,0) = 0, \quad \Theta(\xi,0) = 1$$

$$f'(\xi,\infty) = 0, \quad \Theta(\xi,\infty) = 0 \qquad (17)$$

In the light of the previous investigations, we noted that in the absence of thermal radiation (R_d=0) Hunt and Wilks [11] discussed the above posed problem (15)-(17) with constant transverse magnetic field i.e. by taking $B_{\bar{y}} = B_0$. In this investigation, full numerical solutions via algorithm based upon Keller box technique had been obtained for the whole range of local Hartmann parameter, ξ. However, Hunt and Wilks [11] also obtained the asymptotic solutions for the regions where ξ is treated to be sufficiently large and small.

The formulation obtained in (15)-(17) has been integrated numerically via Keller-box scheme, introduced by Keller and Cebeci [12].

Now, one can obtain the local skin friction coefficient, $C_{f\lambda}Gr_\lambda^{-3/4}$, and the local Nusselt number, $Nu_\lambda Gr_\lambda^{-1/4}$, from the following relations:

$$C_{f\lambda}Gr_\lambda^{-3/4} = X^{1/4}(1+X)^{-3/4} f''(0,X), \qquad (18)$$

$$Nu_\lambda Gr_\lambda^{-1/4} = -\left(1 + \frac{4}{3}R_d\theta_w^3\right) X^{1/4}(1+X)^{-1/4}\Theta'(0,X)$$

4. RESULTS AND DISCUSSION

Present analysis deals with the free convection flow of electrically conducting and optically dense gray fluid over a vertical plate in the presence of sinusoidal magnetic field. Solutions of the governing equations are obtained by two numerical techniques for the entire range of X, namely, (i) direct numerical simulation (DNS) method and (ii) Keller-box scheme (which is applied after reducing boundary layer equations into non-similar equations with the help of SFF). It should be noted that in the present investigation the value of Prandtl number, Pr, is taken to be 0.05 (that is appropriate for lithium). Numerical values of local skin-friction coefficient, $C_{f\lambda}Gr_\lambda^{-3/4}$, and local Nusselt number, $Nu_\lambda Gr_\lambda^{-1/4}$ obtained by DNS are entered, respectively, in Table 1 against X [0.01,10.0] for R_d=0.0 and 2.0 while values of other parameters are taken as θ_w=2.1, M=0.5 and Pr=0.05 and compared quantitatively with those obtained from stream function formulation (SFF). It is inferred from these numerical values that local skin friction coefficient and local Nusselt number coefficient enhances considerably, owing to increase in the thermal radiation parameter, R_d. Due to the intense thermal radiation inside the boundary layer, rate of energy

transport of the fluid is increased sharply which in turn gives rise to the temperature of the fluid in the surrounding of the plate surface. Although the comparison shows good agreement between the two methods, but, in view of computational time and accuracy, SFF sounds to be more economical as compared to DNS method.

Table 1: Numerical values of coefficient of skin-friction with R_d = 0.0, 2.0 and θ_w =2.1, Pr=0.05 and M= 0.5

X	$C_{f\lambda}Gr_\lambda^{-3/4}$			
	$R_d = 0.0$		$R_d = 2.0$	
	DNS	SFF	DNS	SFF
0.01	0.3125	0.4055	0.3610	0.4575
0.2	0.8216	0.8383	0.9359	0.9454
0.5	0.9392	0.9522	1.0618	1.0677
1.0	1.1983	1.2023	1.3431	1.3382
2.0	1.3648	1.3680	1.5240	1.5184
3.0	1.4605	1.4645	1.6270	1.6217
4.0	1.5264	1.5316	1.6967	1.6920

Table 2: Numerical values of coefficient of Nusselt number with R_d = 0.0, 2.0 and θ_w =2.1, Pr=0.05 and M= 0.5

X	$Nu_\lambda Gr_\lambda^{-1/4}$			
	$R_d = 0.0$		$R_d = 2.0$	
	DNS	SFF	DNS	SFF
0.01	0.6181	0.3808	2.7330	1.9425
0.2	0.1798	0.1772	0.7770	0.8929
0.5	0.1313	0.1306	0.5782	0.6669
1.0	0.1164	0.1159	0.5151	0.5730
2.0	0.0937	0.0924	0.4144	0.4485
3.0	0.0838	0.0819	0.3661	0.3843
4.0	0.0773	0.0749	0.3360	0.3459

Further, we discuss the results in terms of coefficient of local skin friction, $C_{f\lambda}Gr_\lambda^{-3/4}$ and coefficient of local Nusselt number, $Nu_\lambda Gr_\lambda^{-1/4}$ for different values of the physical parameters, *i.e.*, for magnetic field parameter (or Hartmann number), M, thermal radiation parameter (or Planck constant), R_d and surface temperature parameter, θ_w against the dimensionless axial distance, X.

4.1 EFFECT OF PHYSICAL PARAMETERS *M* AND θ_w ON $C_{f\lambda}Gr_\lambda^{-3/4}$ AND $Nu_\lambda Gr_\lambda^{-1/4}$

Firstly, the influence of magnetic field parameter, M = 0.0, 0.5, 1.0 and 2.0 is illustrated in Fig. 2 for R_d =2.0, θ_w = 2.1 and Pr = 0.05 on coefficient of local skin friction, $C_{f\lambda}Gr_\lambda^{-3/4}$, and coefficient of local Nusselt number, $Nu_\lambda Gr_\lambda^{-1/4}$. In can be seen from these figures that both coefficients of local skin friction and local Nusselt number decreases considerably due to the increase in magnetic field parameter which acts as retarding force. Intense amount of magnetic field inside the boundary

layer literally increases the Lorentz force which significantly opposes the flow in the reverse direction. Thus, coefficients of local skin friction and local Nusselt number diminish. In these figures one can see that amplitude of the flow pattern increases considerably as M is intensified, which is expected since the waves build each other due to the constructive interferences. Further, in Fig. 2(b) it is observed that the sinusoidal waves decay down smoothly as the fluid moves away from the leading edge of the plate, which signifies that gradually the sinusoidal flow patterns settle down to their asymptotic values.

The variation of local skin friction coefficient $C_{f\lambda}Gr_\lambda^{-3/4}$ and local Nusselt number coefficient $Nu_\lambda Gr_\lambda^{-1/4}$ is inspected for θ_w = 1.1, 2.1 and 3.1 while R_d = 2.0, M=0.5 and Pr = 0.05 in Fig. 3. Here, notable enhancement is recorded in the coefficient of local skin friction and the local Nusselt number coefficients as we
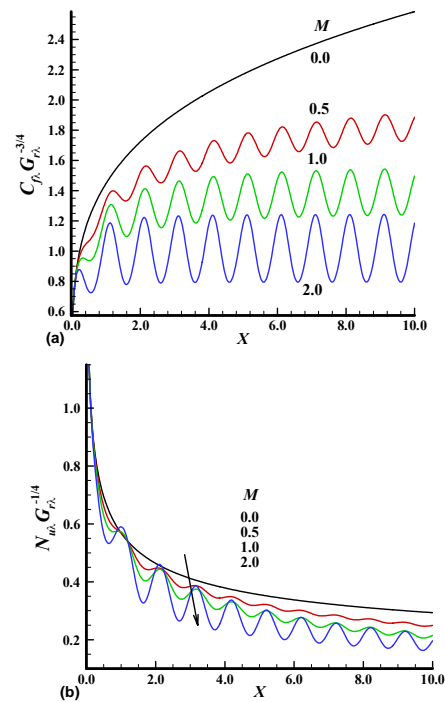

Fig 2 (a) Variation of coefficient of local skin friction and (b) coefficient of local Nusselt number with X for M = 0.0, 0.5, 1.0, 2.0 while R_d = 2.0, θ_w = 2.1 and Pr = 0.05

intensify the surface temperature parameter θ_w Such a behavior is possible because increment in surface temperature parameter, θ_w leads to an increase in the motion of the fluid particles. This rapid increase in the motion of the fluid accelerates the flow rate near the plate. Thus, surface temperature parameter is responsible for the raise in the temperature of the fluid and ultimately wall shear stress and heat transfer rate increases. Further, impact of surface temperature parameter tends to decrease the momentum boundary layer thickness whereas thermal boundary layer thickness increases a little.
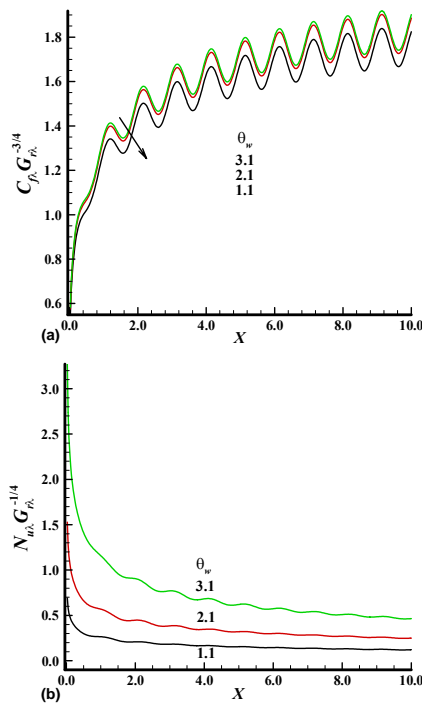
Fig 3. (a) Variation of coefficient of local skin friction and (b) coefficient of local Nusselt number with X for θ_w = 1.1, 2.1, 3.1 while R_d = 2.0, M = 0.5 and Pr = 0.05.

4.2. EFFECT ON *M* ON STREAMLINES AND ISOTHERMLINES

We now focus our attention to observe the effects of magnetic field parameter, M on stream lines and isotherm lines for the case of liquid metals.

The influence of magnetic field on streamlines and isotherms with effect of thermal radiation parameter (for R_d =2.0) is depicted in Figs. 4 and 5. In these figures magnetic field parameter is chosen to be 2.0 and 4.0 whilst Prandtl number, Pr, is 0.05 and surface temperature parameter, θ_w = 2.1. From the Figs. 4(a) and 4(b) it is retrieved that ψ_{max} diminishes steadily from 21.3 to 15.5 for the increasing values of magnetic field parameter. Again, this indicates that magnetic field opposes the flow, as it is noticed earlier. Basically Figs. 4-5 are drawn to show how the velocity and temperature of the flow becomes stronger because of the response gained due to the presence of thermal radiation. For $R_d >$ 0, fluid energy increases rapidly as the fluid molecules are heated up sharply which conclusively give rise to the temperature of the fluid in the neighborhood of the surface of the plate that leads to increase in the temperature gradient. Thus due to the increment of energy at the molecular level, the fluid moves more frequently and in turn increase the momentum as well as thermal boundary layer thicknesses.
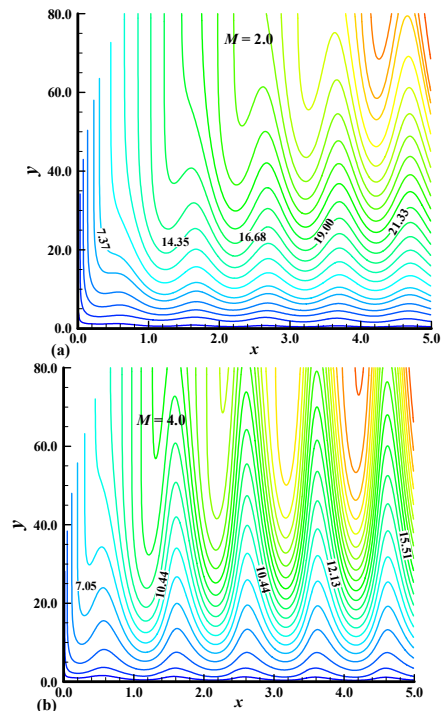

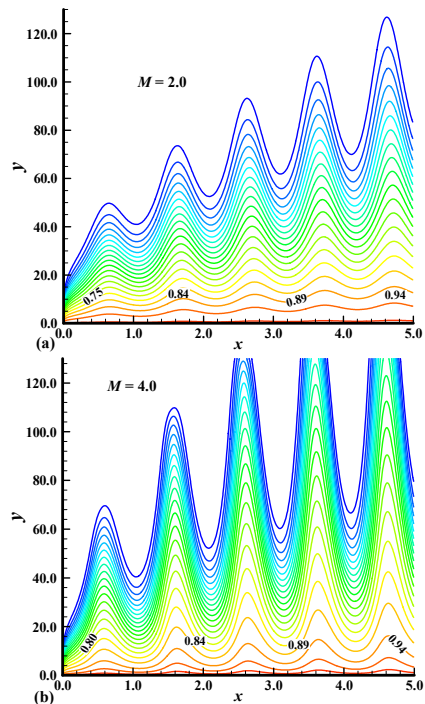Fig 4. Streamlines for (a) M = 2.0 (b) M = 4.0 while θ_w = 2.1, R_d = 2.0 and Pr = 0.05.

Fig 5. Isotherm lines for (a) M = 2.0 (b) M = 4.0 while θ_w = 2.1, R_d = 2.0 and Pr = 0.05.

5. CONCLUSIONS

In this paper, the interaction of magnetohydrodynamic and thermal radiation on the two dimensional free convection flow of an electrically

conducting and optically dense gray viscous fluid over a semi-infinite vertical flat surface has been studied. For entire range of locally varying parameter X, the governing equations are reduced to parabolic partial differential equations using SFF which are then integrated numerically by employing the Keller-box method. Further, DNS technique is also applied to obtain the solution of the boundary layer problem and results obtained are compared quantitatively with the former method. The numerical values thus obtained in terms of local skin friction coefficient $C_{fx}Gr_\lambda^{-3/4}$ and local Nusselt number, $Nu_\lambda Gr_\lambda^{-1/4}$ for various values of the physical parameters for low Prandtl number. From the above investigation it may conclude that (i) Coefficient of local skin friction and coefficient of local Nusselt number diminishes owing to the increase in the magnetic field parameter M, (iii) increase of surface temperature parameter is responsible for the upraise in the temperature of the fluid and ultimately it leads to increase wall shear stress and heat transfer rate and (iv) it is noted that coefficient of local skin friction and coefficient of local Nusselt number diminishes due to the retarding effects of magnetic field on the fluid flow.

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